A Mechanistic Model for Two-Phase Annular-Mist Flow in Vertical Pipes

The physical phenomena associated with two-phase annular-mist flow in vertical pipes are considered. A new mechanistic model is developed to predict liquid entrainment, liquid film thickness, *in situ* velocities, and pressure drop. The model is compared to field data and found to be superior to the three commonly used empirical correlations.

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Introduction

Gas wells that produce liquids can experience liquid loading, which increases the pressure drop and thus restricts reservoir drawdown and reduces production. Liquid loading occurs when the flowing gas velocity is insufficient to surface the liquids that are produced with the gas or condensate in the tubing as the pressure and temperature decrease. As liquids accumulate in the lower portion of the tubing, the back pressure on the formation increases, decreasing the production rate. The back pressure on the formation may reach a critical pressure at which the well dies. All gas wells that produce liquids will experience liquid loading with reservoir depletion. Liquid loading problems may be eliminated by reducing the tubing diameter and/or the surface pressure. Both these techniques increase the gas velocity in the well. The prediction and analysis of liquid loading require multiphase flow equations for determining the flow regime and tubing pressure as a function of gas rate, gas liquid ratio, well diameter, well depth, surface pressure, and fluid properties.

The purpose of this paper is to formulate and develop a new mechanistic model for two-phase annular-mist flow in vertical pipes that permits calculation of liquid entrainment, average liquid film thickness, *in situ* velocities, and pressure drop.

Mathematical Model

The mechanistic model for vertical annular-mist flow is formulated based on the assumption that the flow is fully developed and stable. It is further assumed that at any given location in the tubing, the fraction of entrained liquid is uniformly dispersed as mist in the gas core. The rest of the liquid resides in a film of uniform thickness, δ, around the pipe wall. The idealized flow configuration is shown in Figure 1.

When gas and liquid flow concurrently upward at high gas rate and low liquid rate, annular-mist flow occurs. Taitel et al. (1980) have developed a relation which predicts that annular-mist flow exists if

$$V_{sg^*} \geq 3.1 \left[\frac{\sigma\, g(\rho_L - \rho_g)}{\rho_g^2}\right]^{1/4} \tag{1}$$

This flow regime transition for annular-mist flow is based on the balance between gravity and drag forces acting on spherical droplets. Although this equation has been criticized by Chen and Spedding (1983), Sylvester (1984) has shown it to be the most realistic of the available flow pattern map models, especially for high-pressure gas-condensate systems.

During annular-mist flow a fraction of the flowing liquid will be entrained or dispersed in the gas stream while the rest is confined to a thin liquid film on the pipe wall. Wallis (1969) proposed an expression for the fraction of the liquid entrained, F_E, where F_E, as shown below, is dependent upon the superficial gas velocity, gas viscosity, gas density, liquid density, and interfacial tension.

$$F_E = 1 - e^{-0.125(\beta - 1.5)} \tag{2}$$

where

$$\beta = \frac{3048\, V_{sg}\mu_g}{\sigma} (\rho_g/\rho_L)^{1/2} \tag{3}$$

This expression predicts that the liquid fraction entrained increases as the superficial gas velocity, gas viscosity, or gas density increases or as the liquid density or interfacial tension decreases. Knowing the fraction of the liquid entrained permits the actual mixture density of the gas core to be calculated from the relation

$$\rho_m = \lambda \rho_L + (1 - \lambda)\, \rho_g \tag{4}$$

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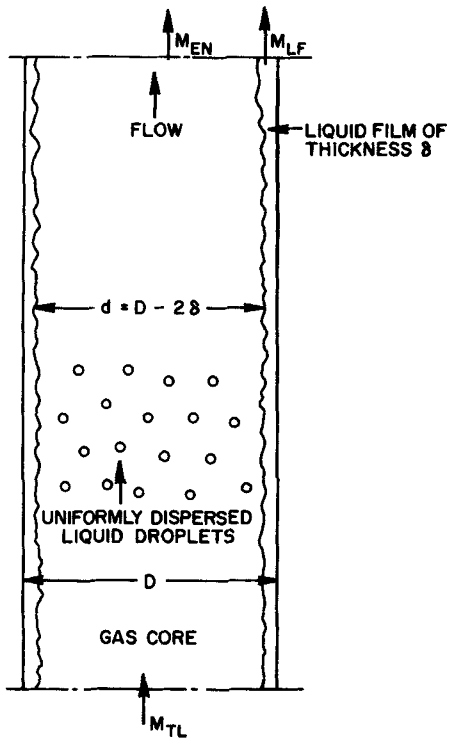

Figure 1. Idealized flow configuration of vertical annular-mist flow model.

where the *in situ* liquid holdup, λ, is given by

$$\lambda = \frac{F_E\, q_L}{(F_E\, q_L + q_g)} \tag{5}$$

The existence of the annular liquid film on the pipe wall reduces the actual pipe cross-sectional area, which causes the gas velocity to be greater than the superficial gas velocity. In addition, since the liquid film surface is wavy it creates a rough surface over which the gas core must flow. The roughness increases the loss due to frictional pressure.

The pressure gradient due to friction may be written

$$\left(\frac{dp}{dz}\right)_f = \frac{\rho_m f_t V_m^2}{2D} \tag{6}$$

where f_t is the friction factor and V_m is the mixture velocity given by

$$V_m = M_{LG}/\rho_m A_c \tag{7}$$

where M_{LG}, the mixture mass flow rate in the gas core, is given by

$$M_{LG} = M_{EN} + M_G = F_E\, M_{TL} + M_G \tag{8}$$

and A_c, the cross-sectional area of the gas core, is given by

$$A_c = \frac{\pi(D - 2\delta)^2}{4} = \frac{\pi d^2}{4} \tag{9}$$

The friction factor is determined from the modified Zigrang-Sylvester (1982) equation

$$\frac{1}{\sqrt{f_t}} = -2.0 \log\left[\frac{\epsilon/D}{3.7} - \frac{5.02}{Re_m}\log\left(\frac{\epsilon/D}{3.7} + \frac{13}{Re_m}\right)\right] \tag{10}$$

where the mixture Reynolds number is given by

$$Re_m = \frac{D\rho_m V_m}{\mu_m} \tag{11}$$

the mixture viscosity by

$$\mu_m = \lambda\, \mu_L + (1-\lambda)\mu_g \tag{12}$$

and ϵ/D is the relative effective roughness for annular-mist flow. This roughness is taken to be the ratio of the time-averaged thickness of the annular liquid film to the pipe diameter. Henstock and Hanratty (1976) determined this ratio to be

$$\frac{\epsilon}{D} = \frac{6.59F}{(1 + 1{,}400F)^{1/2}} = \frac{\delta}{D} \tag{13}$$

where

$$F = \frac{\{[0.707(Re_L)^{0.5}]^{2.5} + [0.0379(Re_g)^{0.9}]^{2.5}\}^{0.4}}{(Re_g)^{0.9}(\mu_g/\mu_L)(\rho_L/\rho_g)^{0.5}} \tag{14}$$

and

$$Re_L = \frac{4M_{LF}}{\mu_L \pi D} \tag{15}$$

$$Re_g = \frac{\rho_g D V_{sg}}{\mu_g} \tag{16}$$

The total pressure gradient consists of an elevation term $(dp/dh)_E$, a friction term $(dp/dh)_f$, and an acceleration term $(dp/dh)_A$, and can be written in the form

$$\frac{dp}{dz} = \left(\frac{dp}{dz}\right)_E + \left(\frac{dp}{dz}\right)_f + \left(\frac{dp}{dz}\right)_A \tag{17}$$

The elevation term is given by

$$\left(\frac{dp}{dz}\right)_E = \rho_m \tag{18}$$

The friction term was given by Eq. 6, and the acceleration term is

$$\left(\frac{dp}{dz}\right)_A = \frac{\rho_m V_m dV_m}{dz} \tag{19}$$

Computer Program

Since this new model requires an iterative trial-and-error calculation, a computer program was developed to calculate surface pressure given bottomhole conditions, and vice versa. The computer program requires the input data listed in Table 1. In

Table 1. Required Input Data

Well depth, H	Bottomhole temp., T_B
Tubing dia., D	Surface temp., T_{SUR}
Tubing roughness*, ϵ	Flowing bottomhole press., P_{wf}
Liquid rate, Q_L	Flowing surface press., P_{SUR}
Gas rate, Q_g	

*Tubing roughness is needed only if all liquid is entrained and/or a single-phase flow pressure calculation is to be made.

addition, the fluid property information shown in Table 2 must be known as a function of temperature and pressure. The calculational procedure is summarized in Table 3. The computer program consists of a main program and fourteen subprograms.

Comparison with Field Data

The effectiveness of the new model was evaluated by comparing its predictions with field data. It is important to note that the new model or any model may be limited by the accuracy of the PVT correlations used.

Water-gas data were taken from Camacho (1970) and Reinicke and Remer (1984). Five cases from Reinicke and Remer were not included in the analysis because the inclination from the vertical was significantly more than 10°. Although the new model was developed for vertical flow, 70 field data points with inclinations less than 10° were included in the analysis. The new model was also tested against 93 gas-oil data points (Govier and Fogarasi, 1975).

A summary of the statistical results is shown in Table 4. All cases tested converged except for one oil-gas data point. There were eight data points for water-gas flow and four for oil-gas flow in which the pressure drop was significantly overpredicted. All these had very high gas velocities (>8 m/s). A possible explanation for this may be that the Henstock and Hanratty (1976) correlation overpredicts the effective relative roughness at high gas velocity. If this occurs, there is a spiral effect that overpredicts pressure drop and gas velocity. The Henstock and

Table 2. Required Fluid Property Data

Liquid density, ρ_L
Gas density, ρ_g
Liquid viscosity, μ_L
Gas viscosity, μ_g
Liquid-gas interfacial tension, σ

Table 3. Summary of Calculational Procedure

Calculate the following at bottomhole temperature and pressure

1. From fluid property correlations calculate solution gas-oil ratio, oil formation volume factor, water formation volume factor, oil-gas interfacial tension, gas compressibility factor
2. Calculate gas and liquid densities and interfacial tension
3. Calculate gas and liquid volumetric flow rates
4. Calculate total gas and liquid mass flow rates
5. Calculate superficial gas velocity
6. Compare superficial gas velocity with Eq. 1 to determine flow regime
7. If annular-mist flow exists, select number of length increments and calculate length increment, ΔH
8. Estimate pressure drop ΔP corresponding to calculated ΔH
9. Calculate average temperature
10. Calculate average pressure
11. Calculate the following at average temperature and pressure
    a. From fluid property correlations recalculate fluid property values in steps 1 and 2
    b. Gas and liquid volumetric flow rates
    c. Superficial gas and liquid velocities
    d. Fraction of liquid entrained
    e. Mass flow rate of liquid entrained
    f. Mixture mass flow in gas core
    g. Volumetric flow rate of entrained liquid
    h. Liquid holdup
    i. Effective average liquid film thickness
    j. Actual area and gas velocity
    k. Mixture viscosity
    l. Mixture density
    m. Mixture velocity
    n. Mixture Reynolds number
    o. Liquid and gas Reynolds numbers
    p. Pseudorelative roughness
    q. Two-phase friction factor
    r. Friction gradient
    s. Elevation gradient
    t. Total pressure gradient
12. Compute ΔP from the relation $\Delta P = (dp/dz)(\Delta z)$
13. If calculated and estimated ΔP values are not within selected tolerance, take calculated ΔP as a new estimate and repeat steps 10–12 until convergence is obtained
14. Compute the next pressure and temperature increment:

$$P_{i+1} = P_i - \Delta P$$

$$T_{i+1} = T_i - \frac{(T_B - T_{SUR})}{H} \Delta z$$

15. Repeat steps 8–14 until total number of increments is reached

Table 4. Comparison of Empirical Correlations and New Model

Correlations or Model	No. of Cases	Avg. % Diff.	Abs. Avg. % Diff.	Std. Dev.	Corr. Coeff.	No. of Cases that Failed to Converge
		Water-Gas Flow Data				
Present work	119	−2.48	13.27	21.79	0.994	0
Hagedorn & Brown (1965)	118	9.21	13.86	16.66	0.9927	1
Dun & Ross (1963)	119	25.05	27.60	24.50	0.9856	0
Beggs & Brill (1973)	110	22.94	25.29	18.00	0.9901	9
		Oil-Gas Flow Date				
Present work	92	2.12	8.37	13.47	0.973	1
Hagedorn & Brown (1965)	93	8.45	10.20	9.93	0.9949	0
Dun & Ros (1963)	93	−3.30	14.83	19.24	0.9837	0
Beggs and Brill (1973)	76	20.04	20.16	13.78	0.9680	17

Hanratty correlation is based on water-air flow data obtained over a limited experimental range ($Re_g \leq 260{,}000$). The field data had calculated gas Reynolds numbers (Re_g) significantly higher than 260,000. In fact, almost all Re_g calculated were over 1,000,000. This extrapolation of the Henstock-Hanratty correlation could have an effect on the accuracy of the effective relative roughness prediction at high gas velocity.

There were fifteen points for water-gas flow and two points for the oil-gas flow data that according to the Taitel et al. (1980) flow regime criteria were not in annular-mist flow at the bottomhole conditions. In addition, there were twelve points for water-gas flow that did not attain annular-mist flow over the entire tubing length. For these cases and some of the cases that achieved annular-mist flow part way up the tubing, most pressure drops were underpredicted. These underpredictions are not surprising because these cases were in the slug or churn flow regimes, which have pressure gradients significantly higher than annular-mist flow.

As shown in Table 4, the new model has a low average percent difference, and a low average absolute percentage difference. Furthermore, the calculated correlation coefficients for both water-gas and oil-gas flow were greater than 0.97, which certainly suggests that there is a very strong linear relationship between the calculated and field wellhead pressures. Figures 2 and 3 show that the "best" least-squares straight, broken lines were quite close to the ideal straight lines.

Comparison with Available Empirical Correlations

Three commonly used empirical correlations were utilized to test the predictive accuracy of the new model. These two-phase empirical correlations are those of Hagedorn and Brown (1965), Duns and Ros (1963), and Beggs and Brill (1973).

Table 4 summarizes the statistical results of the comparison of the new model with the three empirical correlations. The new model outperforms each of these correlations. The calculated average percent differences and absolute average percent differences for the new model are consistently lower than the other correlations.

The predictive accuracy of the new model for pressures in two-phase annular-mist flow in vertical pipes is good and is superior to the three commonly used empirical correlations. The new model could be improved if better correlations were available for effective film thickness and liquid entrainment.

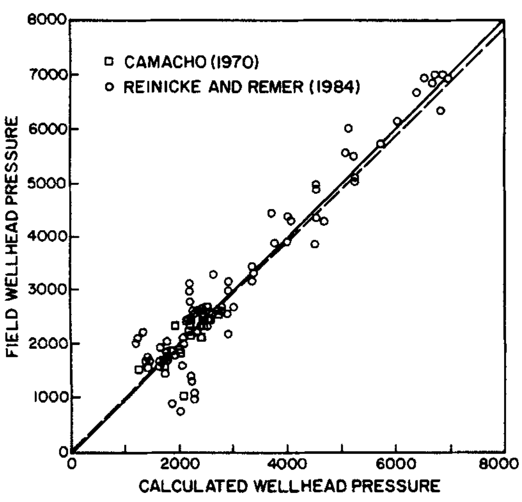

Figure 2. Comparison of model prediction with experimental data.

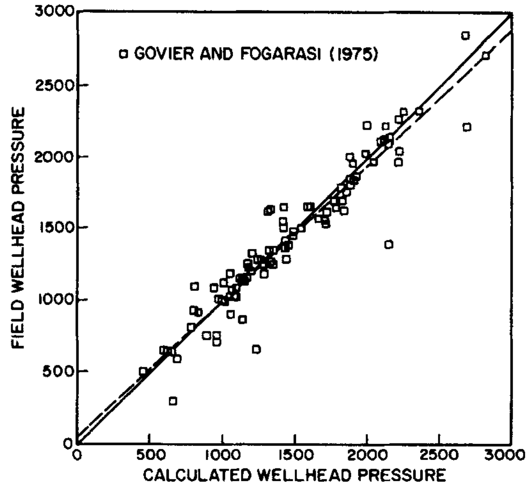

Figure 3. Comparison of model prediction with experimental data.

Notation

A_c = core cross-sectional area
d = core diameter
D = tubing diameter
f_t = two-phase friction factor
F = Henstock-Hanratty dimensionless group, Eq. 14
F_E = fraction of liquid entrained
g = acceleration due to gravity
h = depth increment
H = well depth
M_G = gas mass flow rate
M_{EN} = mass flow rate of entrained liquid
M_{LF} = mass flow rate of annular liquid film
M_{LG} = mixture mass flow rate in core
M_{TL} = total liquid mass flow rate
p = pressure
P_{SUR} = wellhead pressure
P_{wf} = flowing bottomhole pressure
q_g = *in-situ* gas volumetric flow rate
q_L = *in-situ* liquid volumetric flow rate
Q_L = volumetric flow rate of liquid
Q_g = volumetric flow rate of gas
Re_g = gas Reynolds number
Re_L = liquid Reynolds number
Re_m = mixture Reynolds number
T_B = bottomhole temperature
T_{SUR} = surface temperature
V_m = *in-situ* mixture velocity
V_{sg} = *in-situ* superficial velocity gas velocity
V_{sg^*} = superficial gas velocity required for annular-mist flow
Z = well depth

Greek letters

β = dimensionless quantity in Wallis correlation, Eq. 3
δ = average effective film thickness
ϵ = absolute roughness
ρ_g = density of gas
ρ_L = density of liquid
ρ_m = density of mixture
σ = liquid-gas interfacial tension
μ_g = gas viscosity
μ_L = liquid viscosity
μ_m = mixture viscosity
λ = *in-situ* liquid core holdup

Literature cited

Beggs, H. D., and J. P. Brill, "A Study of Two-Phase Flow in Inclined Pipes," *J. Pet. Technol.*, **25**, 607 (1973).

Camacho, C. A., "Comparison of Correlations for Predicting Pressure Losses in High Gas-Liquid Ratio Vertical Wells," M.S. Thesis, Univ. Tulsa (1970).

Chen, J. J., and P. L. Spedding, "Transition to Annular Flow in Vertical Upward Gas-Liquid Flow," *AIChE J.*, **29**, 525 (1983).

Duns, H., and N. C. J. Ros, "Vertical Flow of Gas-Liquid Mixtures in Wells," *Proc. 6th World Pet. Cong.*, 451 (1963).

Govier, G. W., and M. Fogarasi, *"Pressure Drop in Wells Producing Gas and Condensate," Canad. J. Pet. Tech.*, 28 (1975).

Hagedorn, A. R., and K. E. Brown, "Experimental Study of Pressure Gradients Occurring During Continuous Two-Phase Flow in Small-Diameter Vertical Conduits," *J. Pet. Technol.*, **17** 475 (Apr., 1965).

Henstock, W. H., and T. J. Hanratty, "The Interfacial Drag and the Height of the Wall Layer in Annular Flows," *AIChE J.*, **22**, 990 (1976).

Reinicke, K. M., and R. J. Remer, "Comparison of Measured and Predicted Pressure Drops in Tubing for High-Water-Cut Gas Wells," SPE Paper No. 13279, Soc. Pet. Eng. 59th Ann. Meet., Houston, (Sept., 1984).

Sylvester, N. D., "Transition to Annular Flow in Vertical Upward Gas-Liquid Flow," *AIChE J.*, **30**, 700 (1984).

Taitel, Y., D. Bornea, and A. E. Dukler, "Modeling Flow Pattern Transitions for Steady Upward Gas-Liquid Flow in Vertical Tubes," *AIChE J.*, **26**, 345 (1980).

Wallis, G. B., *One-Dimensional Two-Phase Flow*, McGraw-Hill, New York, 393 (1969).

Zigrang, D., and N. D. Sylvester, "Explicit Approximation to the Solution of Colebrook's Friction Factor Equation," *AIChE J.*, **28**, 514 (1982).

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